Calculation of observed system response for examples 2A and 2B

Contents

1 Introduction

This document describes the information that must be provided by the user to define the observed system response in the example contained in MATLAB scripts `DataFusionSoftware_2A.m` and `DataFusionSoftware_2B.m`.

In particular, section 2 contains a list of all the numerical input that the user is expected to provide. For users who are interested, section 3 lists previously calculated information that is used when determining the observed system response, while section 4 provides a full mathematical description of how the observed system response values are calculated by the functions `SecondOrderSystemObs.m` and `GenerateSystemResponse.m`.

2 User input

The observed system response is defined by the following information provided by the user:

Observable name and unit

- (optional) the name of the observable, e.g., 'Pressure' [cell B4],
- (optional) the unit of the observable, e.g., 'MPa' [cell B5].

If provided by the user, the name and unit will be used on the figures generated when running the software.

System parameters

- f_{r}, the resonant frequency (in Hz) [cell C9],
- Q_{r}, the Q-factor [cell D9].

3 Additional input

The following information, previously calculated, is used:

- $t_{1,i_1},\ i_1 = 1,\ldots,m_1$, the times at which the 'true' values of the measurand are calculated (corresponding to the sampling frequency f_1),
- $y_{1,i_1},\ i_1 = 1,\ldots,m_1$, the 'true' values of the measurand.

4 Observed system response values

The system response is proportional to $y_{\mathrm{O}}(t)$ that satisfies

$$\frac{\mathrm{d}^2 y_{\mathrm{O}}(t)}{\mathrm{d}t^2} + \alpha_1 \frac{\mathrm{d}y_{\mathrm{O}}(t)}{\mathrm{d}t} + \alpha_0 y_{\mathrm{O}}(t) = y_1(t).$$

This model can be approximated by a linear constant-coefficient difference equation

$$y_{\mathrm{O},i_1+2} + a_0 y_{\mathrm{O},i_1} + a_1 y_{\mathrm{O},i_1+1} = b_0 y_{1,i_1} + b_1 y_{1,i_1+1} + b_2 y_{1,i_1+2},$$

in which

$$a_0 = \frac{c_0\,(\delta t)^2 - 4c_1\delta t + 4}{c_0\,(\delta t)^2 + 4c_1\delta t + 4}, \quad a_1 = \frac{2c_0\,(\delta t)^2 - 8}{c_0\,(\delta t)^2 + 4c_1\delta t + 4},$$

$$b_0 = \frac{(\delta t)^2}{c_0\,(\delta t)^2 + 4c_1\delta t + 4}, \quad b_1 = \frac{2\,(\delta t)^2}{c_0\,(\delta t)^2 + 4c_1\delta t + 4}, \quad b_2 = \frac{(\delta t)^2}{c_0\,(\delta t)^2 + 4c_1\delta t + 4},$$

where

$$\omega_{\mathrm{r}} = 2\pi f_{\mathrm{r}}, \quad d_{\mathrm{r}} = \frac{-\omega_{\mathrm{r}}}{2Q_{\mathrm{r}}}, \quad c_0 = \omega_{\mathrm{r}}^2 + d_{\mathrm{r}}^2, \quad c_1 = -d_{\mathrm{r}} \quad \text{and} \quad \delta t = 1/f_1.$$

The difference equation model may be equivalently expressed as

$$\boldsymbol{x}_{i_1+1} = \boldsymbol{A}\boldsymbol{x}_{i_1} + \boldsymbol{B}y_{1,i_1}, \quad y_{\mathrm{O},i_1+1} = \boldsymbol{C}\boldsymbol{x}_{i_1+1} + \boldsymbol{D}y_{1,i_1+1},$$

where $\boldsymbol{x}_{i_1} = (\xi(t_{i_1}), \xi(t_{i_1+1}))^\top$ is the state vector, and

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 \\ -a_0 & -a_1 \end{bmatrix}, \quad \boldsymbol{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad \boldsymbol{C} = \begin{bmatrix} (b_0 - a_0 b_2) & (b_1 - a_1 b_2) \end{bmatrix}, \quad \boldsymbol{D} = [\ b_2\ ].$$