Calculation of estimates of measurand for example 3B

Contents

1 Introduction

This document describes the information that must be provided by the user to define the sensor responses to the measurand in the example contained in MATLAB script `DataFusionSoftware_3B.m`.

In particular, section 2 contains a list of all the numerical input that the user is expected to provide. For users who are interested, section 3 lists previously calculated information that is used when determining the sensor responses, while sections 4 and 5 provide full mathematical descriptions of how the sensor response values and estimates of the measurand (and associated uncertainties) are calculated, respectively, by the function `LinearCalibrationLagComp.m`.

2 User input

The responses of the sensors to the measurand are defined by the following information provided by the user:

Sampling and quantization

- f_2, the sampling frequency [cell B10],
- (optional) n_{B}, the number of bits for quantization [cell B11],
- (optional) s, the saturation value [cell B12].

It is assumed that $f_2 < f_1$, i.e., the sampling frequency for the sensors is less than the sampling frequency used when calculating the ‘true’ measurand (section 3).

Behaviour of sensors

- n_{S}, the number of sensors [cell B9 – filled in automatically from information in column C],
- $A_{l_{\mathrm{S}}},\ u_{\mathrm{r}}(A_{l_{\mathrm{S}}}),\ l_{\mathrm{S}} = 1,\ldots,n_{\mathrm{S}}$, the estimates of the offset parameter and their associated relative standard uncertainties [columns D and G],
- $B_{l_{\mathrm{S}}},\ u_{\mathrm{r}}(B_{l_{\mathrm{S}}}),\ l_{\mathrm{S}} = 1,\ldots,n_{\mathrm{S}}$, the estimates of the gain parameter and their associated relative standard uncertainties [columns E and H],
- $L_{l_{\mathrm{S}}},\ u_{\mathrm{r}}(L_{l_{\mathrm{S}}}),\ l_{\mathrm{S}} = 1,\ldots,n_{\mathrm{S}}$, the estimates of the time lag and their associated relative standard uncertainties [columns F and J],
- $\mathrm{cov}_r(A_{l_{\mathrm{S}}}, B_{l_{\mathrm{S}}}),\ l_{\mathrm{S}} = 1,\ldots,n_{\mathrm{S}}$, the relative covariances associated with the estimates of the offset and gain parameters [column I],
- $\delta_{l_{\mathrm{S}}},\ l_{\mathrm{S}} = 1,\ldots,n_{\mathrm{S}}$, the levels of additive noise in the sensor output [column K].

Behaviour of faulty sensors

- n_{F}, the number of faulty sensors [cell N9 – filled in automatically from information in column O],
- $I_{\mathrm{F},l_{\mathrm{F}}},\ l_{\mathrm{F}} = 1,\ldots,n_{\mathrm{F}}$, the indices of the faulty sensors [column O],
- $C_{l_{\mathrm{F}}},\ u_{\mathrm{r}}(C_{l_{\mathrm{F}}}),\ l_{\mathrm{F}} = 1,\ldots,n_{\mathrm{F}}$, the estimates of the offset parameter and their associated relative standard uncertainties [columns P and S],
- $D_{l_{\mathrm{F}}},\ u_{\mathrm{r}}(D_{l_{\mathrm{F}}}),\ l_{\mathrm{F}} = 1,\ldots,n_{\mathrm{F}}$, the estimates of the gain parameter and their associated relative standard uncertainties [columns Q and T],
- $M_{l_{\mathrm{F}}},\ u_{\mathrm{r}}(M_{l_{\mathrm{F}}}),\ l_{\mathrm{F}} = 1,\ldots,n_{\mathrm{F}}$, the estimates of the time lag and their associated relative standard uncertainties [columns R and V],
- $\mathrm{cov}_r(C_{l_{\mathrm{F}}}, D_{l_{\mathrm{F}}}),\ l_{\mathrm{F}} = 1,\ldots,n_{\mathrm{F}}$, the relative covariances associated with the estimates of the offset and gain parameters [column U],
- $T^1_{\mathrm{F},l_{\mathrm{F}}},\ l_{\mathrm{F}} = 1,\ldots,n_{\mathrm{F}}$, the times at which sensors begin behaving faultily [column W],
- $T^2_{\mathrm{F},l_{\mathrm{F}}},\ l_{\mathrm{F}} = 1,\ldots,n_{\mathrm{F}}$, the times at which sensors end behaving faultily [column X].

Missing data

- n_{M}, the number of sensors for which packets of data are missing [cell AA9 – filled in automatically from information in column AB],
- n_{D}, the number of data points in each packet [cell AA10],
- $I_{\mathrm{M},l_{\mathrm{M}}},\ l_{\mathrm{M}} = 1,\ldots,n_{\mathrm{M}}$, the indices of the sensors for which packets of data are missing [column AB],

- $p_{\mathrm{M},l_\mathrm{M}},\ l_\mathrm{M} = 1,\ldots,n_\mathrm{M}$, the proportion (expressed as a percentage) of the data packets that are missing within the assigned time interval [column AC],
- $T^1_{\mathrm{M},l_\mathrm{M}},\ l_\mathrm{M} = 1,\ldots,n_\mathrm{M}$, the times at which sensors begin missing packets of data [column AD],
- $T^2_{\mathrm{M},l_\mathrm{M}},\ l_\mathrm{M} = 1,\ldots,n_\mathrm{M}$, the times at which sensors cease missing packets of data [column AE].

3 Additional input

The following information, previously calculated, is used:

- $t_{1,i_1},\ i_1 = 1,\ldots,m_1$, the times at which the 'true' values of the measurand are calculated (corresponding to the sampling frequency f_1),
- $y_{1,i_1},\ i_1 = 1,\ldots,m_1$, the 'true' values of the measurand.

4 Sensor response values

The array $\widetilde{\mathbf{V}}_2$ of sensor response values is given by

$$\widetilde{\mathbf{V}}_2 = [\widetilde{\mathbf{v}}_{2,1} \quad \ldots \quad \widetilde{\mathbf{v}}_{2,n_\mathrm{S}}],$$

where

$$\widetilde{\mathbf{v}}_{2,l_\mathrm{S}} = \begin{bmatrix} \widetilde{v}_{2,l_\mathrm{S},1} \\ \vdots \\ \widetilde{v}_{2,l_\mathrm{S},m_2} \end{bmatrix}$$

is the vector of response values for sensor l_S and is obtained as follows:

1. Draw the time lag $L^*_{l_\mathrm{S}}$ from the rectangular distribution

$$\mathrm{R}\left(L_{l_\mathrm{S}} - \frac{\sqrt{3}u_\mathrm{r}(L_{l_\mathrm{S}})L_{l_\mathrm{S}}}{100},\ L_{l_\mathrm{S}} + \frac{\sqrt{3}u_\mathrm{r}(L_{l_\mathrm{S}})L_{l_\mathrm{S}}}{100}\right).$$

$L^*_{l_\mathrm{S}}$ defines a number $q^*_{l_\mathrm{S}}$ of values of the sampled 'true' measurand, where $q^*_{l_\mathrm{S}}$ is the smallest integer satisfying

$$q^*_{l_\mathrm{S}} \geq f_1 L^*_{l_\mathrm{S}}.$$

Generate the vector $\mathbf{y}^\mathrm{L}_{1,l_\mathrm{S}}$ of time-shifted 'true' measurand values given by

$$y^{\mathrm{L}}_{1,l_{\mathrm{S}},i_1} = \begin{cases} \mathrm{NaN}, & 1 \le i_1 \le q^*_{l_{\mathrm{S}}}, \\ y_{1,i_1-q^*_{l_{\mathrm{S}}}}, & q^*_{l_{\mathrm{S}}}+1 \le i_1 \le m_1, \end{cases}$$

where NaN denotes 'Not a Number'.

Evaluate the vector of sensor responses

$$\mathbf{v}_{1,l_{\mathrm{S}}} = A^{(2)*}_{l_{\mathrm{S}}} + B^{(2)*}_{l_{\mathrm{S}}}\mathbf{y}^{\mathrm{L}}_{1,l_{\mathrm{S}}} + \mathbf{r}_{l_{\mathrm{S}}},$$

corresponding to the times $t_{1,i_1},\ i_1 = 1,\dots,m_1$, where

$$\begin{bmatrix} A^{(2)*}_{l_{\mathrm{S}}} \\ B^{(2)*}_{l_{\mathrm{S}}} \end{bmatrix} \sim \mathrm{N}\left(\begin{bmatrix} A_{l_{\mathrm{S}}} \\ B_{l_{\mathrm{S}}} \end{bmatrix}, \boldsymbol{\Sigma}_{l_{\mathrm{S}}} \right),$$

with

$$\boldsymbol{\Sigma}_{l_{\mathrm{S}}} = \begin{bmatrix} \left(\dfrac{u_{\mathrm{r}}(A_{l_{\mathrm{S}}})A_{l_{\mathrm{S}}}}{100}\right)^2 & \dfrac{\mathrm{cov}_{\mathrm{r}}(A_{l_{\mathrm{S}}},B_{l_{\mathrm{S}}})A_{l_{\mathrm{S}}}B_{l_{\mathrm{S}}}}{100} \\ \dfrac{\mathrm{cov}_{\mathrm{r}}(A_{l_{\mathrm{S}}},B_{l_{\mathrm{S}}})A_{l_{\mathrm{S}}}B_{l_{\mathrm{S}}}}{100} & \left(\dfrac{u_{\mathrm{r}}(B_{l_{\mathrm{S}}})B_{l_{\mathrm{S}}}}{100}\right)^2 \end{bmatrix},$$

and

$$\mathbf{r}_{l_{\mathrm{S}}} = \begin{bmatrix} r_{l_{\mathrm{S}},1} \\ \vdots \\ r_{l_{\mathrm{S}},m_1} \end{bmatrix},$$

with

$$r_{l_{\mathrm{S}},i_1} \sim \mathrm{N}(0,(\delta_{l_{\mathrm{S}}})^2),\ i_1 = 1,\dots,m_1.$$

2. For sensors that are faulty, the response values within the time intervals $\left[T^1_{\mathrm{F},l_{\mathrm{F}}}, T^2_{\mathrm{F},l_{\mathrm{F}}}\right]$ are obtained similarly to those in step 1, but using the parameters $C_{l_{\mathrm{F}}}$, $u_{\mathrm{r}}(C_{l_{\mathrm{F}}})$, $D_{l_{\mathrm{F}}}$, $u_{\mathrm{r}}(D_{l_{\mathrm{F}}})$, $\mathrm{cov}_r(C_{l_{\mathrm{F}}}, D_{l_{\mathrm{F}}})$, $M_{l_{\mathrm{F}}}$ and $u_{\mathrm{r}}(M_{l_{\mathrm{F}}})$, $\quad l_{\mathrm{F}} = 1,\dots,n_{\mathrm{F}}$.

3. Determine the times $t_{2,i_2},\ i_2 = 1,\dots,m_2$, at which the sensor response values are to be evaluated.

4. Evaluate the sensor response values v_{2,l_{S},i_2} corresponding to the times $t_{2,i_2},\ i_2 = 1,\dots,m_2$, by applying linear interpolation to the sensor responses v_{1,l_{S},i_1} corresponding to the times $t_{1,i_1},\ i_1 = 1,\dots,m_1$.

5. The sensor response values $v_{2,l_{\rm S},i_2}$, $i_2 = 1,\ldots,m_2$, are then quantized according to the values of $n_{\rm B}$ and s (if present) to give values

$$\widetilde{v}_{2,l_{\rm S},i_2}, \; i_2 = 1,\ldots,m_2.$$

   If no values have been provided for $n_{\rm B}$ and s, then

$$\widetilde{v}_{2,l_{\rm S},i_2} = v_{2,l_{\rm S},i_2}, \; i_2 = 1,\ldots,m_2.$$

6. For sensors that have missing data, the proportions $p_{{\rm M},l_{\rm M}}$ of data packets (a data packet is defined to be a group of data points that are sequential in time) chosen randomly within the time intervals $\left[T^1_{{\rm M},l_{\rm M}}, T^2_{{\rm M},l_{\rm M}}\right]$ have their values set to NaN, $l_{\rm M} = 1,\ldots,n_{\rm M}$.

5 Measurand estimates and associated uncertainties

Let $q_{l_{\rm S}}$ be the smallest integer satisfying

$$q_{l_{\rm S}} \geq f_2 L_{l_{\rm S}}.$$

The array $\mathbf{Y}_2$ of measurand estimates is given by

$$\mathbf{Y}_2 = \left[\begin{array}{ccc} y_{2,1,1} & \cdots & y_{2,n_{\rm S},1} \\ \vdots & \ddots & \vdots \\ y_{2,1,m_2} & \cdots & y_{2,n_{\rm S},m_2} \end{array}\right],$$

where

$$y_{2,l_{\rm S},i_2} = \begin{cases} \dfrac{\widetilde{v}_{2,l_{\rm S},i_2+q_{l_{\rm S}}} - A_{l_{\rm S}}}{B_{l_{\rm S}}}, & 1 \leq i_2 \leq m_2 - q_{l_{\rm S}}, \\ \text{NaN}, & m_2 - q_{l_{\rm S}} + 1 \leq i_2 \leq m_2. \end{cases}$$

The array $\mathbf{U}_2$ of standard uncertainties associated with the measurand estimates is given by

$$\mathbf{U}_2 = \left[\begin{array}{ccc} u\left(y_{2,1,1}\right) & \cdots & u\left(y_{2,n_{\rm S},1}\right) \\ \vdots & \ddots & \vdots \\ u\left(y_{2,1,m_2}\right) & \cdots & u\left(y_{2,n_{\rm S},m_2}\right) \end{array}\right],$$

where

$$\begin{aligned} u^2\left(y_{2,l_{\rm S},i_2}\right) &= \left(\frac{1}{B_{l_{\rm S}}}\right)^2 \left(\frac{u_{\rm r}(A_{l_{\rm S}})A_{l_{\rm S}}}{100}\right)^2 + \left(\frac{y_{2,l_{\rm S},i_2+q_{l_{\rm S}}}}{B_{l_{\rm S}}}\right)^2 \left(\frac{u_{\rm r}(B_{l_{\rm S}})B_{l_{\rm S}}}{100}\right)^2 + \\ & \quad 2\left(\frac{y_{2,l_{\rm S},i_2+q_{l_{\rm S}}}}{(B_{l_{\rm S}})^2}\right)\left(\frac{\mathrm{cov}_{\rm r}(A_{l_{\rm S}},B_{l_{\rm S}})A_{l_{\rm S}}B_{l_{\rm S}}}{100}\right) + \left(\frac{\delta_{l_{\rm S}}}{B_{l_{\rm S}}}\right)^2, \quad 1 \leq i_2 \leq m_2 - q_{l_{\rm S}}, \end{aligned}$$

and

$$u\left(y_{2,l_{\rm S},i_2}\right) = \text{NaN}, \quad m_2 - q_{l_{\rm S}} + 1 \leq i_2 \leq m_2.$$