Calculation of estimates of measurand for example 3A

Contents

1 Introduction

This document describes the information that must be provided by the user to define the sensor responses to the measurand in the example contained in MATLAB script `DataFusionSoftware_3A.m`.

In particular, section 2 contains a list of all the numerical input that the user is expected to provide. For users who are interested, section 3 lists previously calculated information that is used when determining the sensor responses, while sections 4 and 5 provide full mathematical descriptions of how the sensor response values and estimates of the measurand (and associated uncertainties) are calculated, respectively, by the function `LinearCalibrationLag.m`.

2 User input

The responses of the sensors to the measurand are defined by the following information provided by the user:

Sampling and quantization

- f_2, the sampling frequency [cell B10],

- (optional) n_B, the number of bits for quantization [cell B11],

- (optional) s, the saturation value [cell B12].

It is assumed that $f_2 < f_1$, i.e., the sampling frequency for the sensors is less than the sampling frequency used when calculating the 'true' measurand (section 3).

Behaviour of sensors

- n_S, the number of sensors [cell B9 – filled in automatically from information in column C],

- A_{l_S}, $u_r(A_{l_S})$, $l_S = 1, \ldots, n_S$, the estimates of the offset parameter and their associated relative standard uncertainties [columns D and G],

- B_{l_S}, $u_r(B_{l_S})$, $l_S = 1, \ldots, n_S$, the estimates of the gain parameter and their associated relative standard uncertainties [columns E and H],

- L_{l_S}, $u_r(L_{l_S})$, $l_S = 1, \ldots, n_S$, the estimates of the time lag and their associated relative standard uncertainties [columns F and J],

- $\mathrm{cov}_r(A_{l_S}, B_{l_S})$, $l_S = 1, \ldots, n_S$, the relative covariances associated with the estimates of the offset and gain parameters [column I],

- δ_{l_S}, $l_S = 1, \ldots, n_S$, the levels of additive noise in the sensor output [column K].

Behaviour of faulty sensors

- n_F, the number of faulty sensors [cell N9 – filled in automatically from information in column O],

- I_{F,l_F}, $l_F = 1, \ldots, n_F$, the indices of the faulty sensors [column O],

- C_{l_F}, $u_r(C_{l_F})$, $l_F = 1, \ldots, n_F$, the estimates of the offset parameter and their associated relative standard uncertainties [columns P and S],

- D_{l_F}, $u_r(D_{l_F})$, $l_F = 1, \ldots, n_F$, the estimates of the gain parameter and their associated relative standard uncertainties [columns Q and T],

- M_{l_F}, $u_r(M_{l_F})$, $l_F = 1, \ldots, n_F$, the estimates of the time lag and their associated relative standard uncertainties [columns R and V],

- $\mathrm{cov}_r(C_{l_F}, D_{l_F})$, $l_F = 1, \ldots, n_F$, the relative covariances associated with the estimates of the offset and gain parameters [column U],

- T^1_{F,l_F}, $l_F = 1, \ldots, n_F$, the times at which sensors begin behaving faultily [column W],

- T^2_{F,l_F}, $l_F = 1, \ldots, n_F$, the times at which sensors end behaving faultily [column X].

Missing data

- n_M, the number of sensors for which packets of data are missing [cell AA9 – filled in automatically from information in column AB],

- n_D, the number of data points in each packet [cell AA10],

- I_{M,l_M}, $l_M = 1, \ldots, n_M$, the indices of the sensors for which packets of data are missing [column AB],

- p_{M,l_M}, $l_M = 1, \ldots, n_M$, the proportion (expressed as a percentage) of the data packets that are missing within the assigned time interval [column AC],

- T^1_{M,l_M}, $l_M = 1, \ldots, n_M$, the times at which sensors begin missing packets of data [column AD],

- T^2_{M,l_M}, $l_M = 1, \ldots, n_M$, the times at which sensors cease missing packets of data [column AE].

3   Additional input

The following information, previously calculated, is used:

- t_{1,i_1}, $i_1 = 1, \ldots, m_1$, the times at which the 'true' values of the measurand are calculated (corresponding to the sampling frequency f_1),

- y_{1,i_1}, $i_1 = 1, \ldots, m_1$, the 'true' values of the measurand.

4   Sensor response values

The array $\widetilde{\mathbf{V}}_2$ of sensor response values is given by

$$\widetilde{\mathbf{V}}_2 = \begin{bmatrix} \widetilde{\mathbf{v}}_{2,1} & \ldots & \widetilde{\mathbf{v}}_{2,n_S} \end{bmatrix},$$

where

$$\widetilde{\mathbf{v}}_{2,l_S} = \begin{bmatrix} \widetilde{v}_{2,l_S,1} \\ \vdots \\ \widetilde{v}_{2,l_S,m_2} \end{bmatrix}$$

is the vector of response values for sensor l_S and is obtained as follows:

1. Draw the time lag $L^*_{l_S}$ from the rectangular distribution

$$\mathrm{R}\left(L_{l_S} - \frac{\sqrt{3}u_r(L_{l_S})L_{l_S}}{100}, \ L_{l_S} + \frac{\sqrt{3}u_r(L_{l_S})L_{l_S}}{100}\right).$$

$L^*_{l_S}$ defines a number $q^*_{l_S}$ of values of the sampled 'true' measurand, where $q^*_{l_S}$ is the smallest integer satisfying

$$q^*_{l_S} \geq f_1 L^*_{l_S}.$$

Generate the vector $\mathbf{y}^L_{1,l_S}$ of time-shifted 'true' measurand values given by

$$y^{\mathrm{L}}_{1,l_{\mathrm{S}},i_1} = \begin{cases} \mathrm{NaN}, & 1 \le i_1 \le q^*_{l_{\mathrm{S}}}, \\ y_{1,i_1 - q^*_{l_{\mathrm{S}}}}, & q^*_{l_{\mathrm{S}}} + 1 \le i_1 \le m_1, \end{cases}$$

where NaN denotes 'Not a Number'.

Evaluate the vector of sensor responses

$$\mathbf{v}_{1,l_{\mathrm{S}}} = A^{(2)*}_{l_{\mathrm{S}}} + B^{(2)*}_{l_{\mathrm{S}}} \mathbf{y}^{\mathrm{L}}_{1,l_{\mathrm{S}}} + \mathbf{r}_{l_{\mathrm{S}}},$$

corresponding to the times t_{1,i_1}, $i_1 = 1, \ldots, m_1$, where

$$\begin{bmatrix} A^{(2)*}_{l_{\mathrm{S}}} \\ B^{(2)*}_{l_{\mathrm{S}}} \end{bmatrix} \sim \mathrm{N}\left(\begin{bmatrix} A_{l_{\mathrm{S}}} \\ B_{l_{\mathrm{S}}} \end{bmatrix}, \boldsymbol{\Sigma}_{l_{\mathrm{S}}} \right),$$

with

$$\boldsymbol{\Sigma}_{l_{\mathrm{S}}} = \begin{bmatrix} \left(\dfrac{u_{\mathrm{r}}(A_{l_{\mathrm{S}}}) A_{l_{\mathrm{S}}}}{100} \right)^2 & \dfrac{\mathrm{cov}_{\mathrm{r}}(A_{l_{\mathrm{S}}}, B_{l_{\mathrm{S}}}) A_{l_{\mathrm{S}}} B_{l_{\mathrm{S}}}}{100} \\ \dfrac{\mathrm{cov}_{\mathrm{r}}(A_{l_{\mathrm{S}}}, B_{l_{\mathrm{S}}}) A_{l_{\mathrm{S}}} B_{l_{\mathrm{S}}}}{100} & \left(\dfrac{u_{\mathrm{r}}(B_{l_{\mathrm{S}}}) B_{l_{\mathrm{S}}}}{100} \right)^2 \end{bmatrix},$$

and

$$\mathbf{r}_{l_{\mathrm{S}}} = \begin{bmatrix} r_{l_{\mathrm{S}},1} \\ \vdots \\ r_{l_{\mathrm{S}},m_1} \end{bmatrix},$$

with

$$r_{l_{\mathrm{S}},i_1} \sim \mathrm{N}(0, (\delta_{l_{\mathrm{S}}})^2), \ i_1 = 1, \ldots, m_1.$$

2. For sensors that are faulty, the response values within the time intervals $\left[T^1_{\mathrm{F},l_{\mathrm{F}}}, T^2_{\mathrm{F},l_{\mathrm{F}}} \right]$ are obtained similarly to those in step 1, but using the parameters $C_{l_{\mathrm{F}}}$, $u_{\mathrm{r}}(C_{l_{\mathrm{F}}})$, $D_{l_{\mathrm{F}}}$, $u_{\mathrm{r}}(D_{l_{\mathrm{F}}})$, $\mathrm{cov}_r(C_{l_{\mathrm{F}}}, D_{l_{\mathrm{F}}})$, $M_{l_{\mathrm{F}}}$ and $u_{\mathrm{r}}(M_{l_{\mathrm{F}}})$, $\quad l_{\mathrm{F}} = 1, \ldots, n_{\mathrm{F}}$.

3. Determine the times t_{2,i_2}, $i_2 = 1, \ldots, m_2$, at which the sensor response values are to be evaluated.

4. Evaluate the sensor response values v_{2,l_{S},i_2} corresponding to the times t_{2,i_2}, $i_2 = 1, \ldots, m_2$, by applying linear interpolation to the sensor responses v_{1,l_{S},i_1} corresponding to the times t_{1,i_1}, $i_1 = 1, \ldots, m_1$.

5. The sensor response values v_{2,l_S,i_2}, $i_2 = 1, \ldots, m_2$, are then quantized according to the values of n_B and s (if present) to give values

$$\widetilde{v}_{2,l_S,i_2}, \ i_2 = 1, \ldots, m_2.$$

If no values have been provided for n_B and s, then

$$\widetilde{v}_{2,l_S,i_2} = v_{2,l_S,i_2}, \ i_2 = 1, \ldots, m_2.$$

6. For sensors that have missing data, the proportions p_{M,l_M} of data packets (a data packet is defined to be a group of data points that are sequential in time) chosen randomly within the time intervals $\left[T_{M,l_M}^1, T_{M,l_M}^2 \right]$ have their values set to NaN, $l_M = 1, \ldots, n_M$.

5 Measurand estimates and associated uncertainties

Let q_{l_S} be the smallest integer satisfying

$$q_{l_S} \geq f_2 L_{l_S}.$$

The array $\mathbf{Y}_2$ of measurand estimates is given by

$$\mathbf{Y}_2 = \begin{bmatrix} y_{2,1,1} & \cdots & y_{2,n_S,1} \\ \vdots & \ddots & \vdots \\ y_{2,1,m_2} & \cdots & y_{2,n_S,m_2} \end{bmatrix},$$

where

$$y_{2,l_S,i_2} = \begin{cases} \text{NaN}, & 1 \leq i_2 \leq q_{l_S}, \\ \dfrac{\widetilde{v}_{2,l_S,i_2} - A_{l_S}}{B_{l_S}}, & q_{l_S} + 1 \leq i_2 \leq m_2. \end{cases}$$

The array $\mathbf{U}_2$ of standard uncertainties associated with the measurand estimates is given by

$$\mathbf{U}_2 = \begin{bmatrix} u\left(y_{2,1,1}\right) & \cdots & u\left(y_{2,n_S,1}\right) \\ \vdots & \ddots & \vdots \\ u\left(y_{2,1,m_2}\right) & \cdots & u\left(y_{2,n_S,m_2}\right) \end{bmatrix},$$

where

$$u\left(y_{2,l_S,i_2}\right) = \text{NaN}, \ \ 1 \leq i_2 \leq q_{l_S},$$

and

$$
\begin{aligned}
u^2\left(y_{2,l_S,i_2}\right) = & \left(\frac{1}{B_{l_S}}\right)^2 \left(\frac{u_r(A_{l_S})A_{l_S}}{100}\right)^2 + \left(\frac{y_{2,l_S,i_2}}{B_{l_S}}\right)^2 \left(\frac{u_r(B_{l_S})B_{l_S}}{100}\right)^2 + \\
& 2\left(\frac{y_{2,l_S,i_2}}{(B_{l_S})^2}\right)\left(\frac{\text{cov}_r(A_{l_S}, B_{l_S})A_{l_S}B_{l_S}}{100}\right) + \left(\frac{\delta_{l_S}}{B_{l_S}}\right)^2, \ \ q_{l_S} + 1 \leq i_2 \leq m_2.
\end{aligned}
$$