Calculation of estimates of subsidiary measurand for examples 1A and 1B

Contents

1 Introduction

This document describes the information that must be provided by the user to define the sensor responses to the subsidiary measurand in the examples contained in MATLAB scripts `DataFusionSoftware_1A.m` and `DataFusionSoftware_1B.m`.

In particular, section 2 contains a list of all the numerical input that the user is expected to provide. For users who are interested, section 3 lists previously calculated information that is used when determining the sensor responses, while sections 4 and 5 provide full mathematical descriptions of how the sensor response values and estimates of the subsidiary measurand (and associated uncertainties) are calculated, respectively, by the function `LinearCalibration.m`.

2 User input

The responses of the sensors to the subsidiary measurand are defined by the following information provided by the user:

Sampling and quantization

- $f_2^{(2)}$, the sampling frequency (in Hz) [cell B10],
- (optional) $n_{\mathrm{B}}^{(2)}$, the number of bits for quantization [cell B11],
- (optional) $s^{(2)}$, the saturation value [cell B12].

It is assumed that $f_2^{(2)} < f_1^{(2)}$, i.e., the sampling frequency for the sensors is less than the sampling frequency used when calculating the 'true' subsidiary measurand (section 3).

Behaviour of sensors

- $n_{\mathrm{S}}^{(2)}$, the number of sensors [cell B9 – filled in automatically from information in column C],
- $A_{l_{\mathrm{S}}}^{(2)},\ u_{\mathrm{r}}\left(A_{l_{\mathrm{S}}}^{(2)}\right),\ l_{\mathrm{S}}=1,\ldots,n_{\mathrm{S}}^{(2)}$, the estimates of the offset parameter and their associated relative standard uncertainties [columns D and F],
- $B_{l_{\mathrm{S}}}^{(2)},\ u_{\mathrm{r}}\left(B_{l_{\mathrm{S}}}^{(2)}\right),\ l_{\mathrm{S}}=1,\ldots,n_{\mathrm{S}}^{(2)}$, the estimates of the gain parameter and their associated relative standard uncertainties [columns E and G],
- $\mathrm{cov}_r\left(A_{l_{\mathrm{S}}}^{(2)}, B_{l_{\mathrm{S}}}^{(2)}\right),\ l_{\mathrm{S}}=1,\ldots,n_{\mathrm{S}}^{(2)}$, the relative covariances associated with the estimates of the offset and gain parameters [column H],
- $\delta_{l_{\mathrm{S}}}^{(2)},\ l_{\mathrm{S}}=1,\ldots,n_{\mathrm{S}}^{(2)}$, the levels of additive noise in the sensor output [column I].

Behaviour of faulty sensors

- $n_{\mathrm{F}}^{(2)}$, the number of faulty sensors [cell L9 – filled in automatically from information in column M],
- $I_{\mathrm{F},l_{\mathrm{F}}}^{(2)},\ l_{\mathrm{F}}=1,\ldots,n_{\mathrm{F}}^{(2)}$, the indices of the faulty sensors [column M],
- $C_{l_{\mathrm{F}}}^{(2)},\ u_{\mathrm{r}}\left(C_{l_{\mathrm{F}}}^{(2)}\right),\ l_{\mathrm{F}}=1,\ldots,n_{\mathrm{F}}^{(2)}$, the estimates of the offset parameter and their associated relative standard uncertainties [columns N and P],
- $D_{l_{\mathrm{F}}}^{(2)},\ u_{\mathrm{r}}\left(D_{l_{\mathrm{F}}}^{(2)}\right),\ l_{\mathrm{F}}=1,\ldots,n_{\mathrm{F}}^{(2)}$, the estimates of the gain parameter and their associated relative standard uncertainties [columns O and Q],
- $\mathrm{cov}_r\left(C_{l_{\mathrm{F}}}^{(2)}, D_{l_{\mathrm{F}}}^{(2)}\right),\ l_{\mathrm{F}}=1,\ldots,n_{\mathrm{F}}^{(2)}$, the relative covariances associated with the estimates of the offset and gain parameters [column R],
- $T_{\mathrm{F},l_{\mathrm{F}}}^{(2),1},\ l_{\mathrm{F}}=1,\ldots,n_{\mathrm{F}}^{(2)}$, the times at which sensors begin behaving faultily [column S],
- $T_{\mathrm{F},l_{\mathrm{F}}}^{(2),2},\ l_{\mathrm{F}}=1,\ldots,n_{\mathrm{F}}^{(2)}$, the times at which sensors end behaving faultily [column T].

Missing data

- $n_{\mathrm{M}}^{(2)}$, the number of sensors for which packets of data are missing [cell W9 – filled in automatically from information in column X],
- $n_{\mathrm{D}}^{(2)}$, the number of data points in each packet [cell W10],
- $I_{\mathrm{M},l_{\mathrm{M}}}^{(2)},\ l_{\mathrm{M}}=1,\ldots,n_{\mathrm{M}}^{(2)}$, the indices of the sensors for which packets of data are missing [column X],

- $p_{\mathrm{M},l_\mathrm{M}}^{(2)}$, $l_\mathrm{M} = 1, \ldots, n_\mathrm{M}^{(2)}$, the proportion (expressed as a percentage) of the data packets that are missing within the assigned time interval [column Y],
- $T_{\mathrm{M},l_\mathrm{M}}^{(2),1}$, $l_\mathrm{M} = 1, \ldots, n_\mathrm{M}^{(2)}$, the times at which sensors begin missing packets of data [column Z],
- $T_{\mathrm{M},l_\mathrm{M}}^{(2),2}$, $l_\mathrm{M} = 1, \ldots, n_\mathrm{M}^{(2)}$, the times at which sensors cease missing packets of data [column AA].

3 Additional input

The following information, previously calculated, is used:

- $t_{1,i_1}^{(2)}$, $i_1 = 1, \ldots, m_1^{(2)}$, the times at which the 'true' values of the subsidiary measurand are calculated (corresponding to the sampling frequency $f_1^{(2)}$),
- $y_{1,i_1}^{(2)}$, $i_1 = 1, \ldots, m_1^{(2)}$, the 'true' values of the subsidiary measurand, stored in vector $\mathbf{y}_1^{(2)}$.

4 Sensor response values

The array $\widetilde{\mathbf{V}}_2^{(2)}$ of sensor response values is given by

$$\widetilde{\mathbf{V}}_2^{(2)} = \begin{bmatrix} \widetilde{\mathbf{v}}_{2,1}^{(2)} & \ldots & \widetilde{\mathbf{v}}_{2,n_\mathrm{S}^{(2)}}^{(2)} \end{bmatrix},$$

where

$$\widetilde{\mathbf{v}}_{2,l_\mathrm{S}}^{(2)} = \begin{bmatrix} \widetilde{v}_{2,l_\mathrm{S},1}^{(2)} \\ \vdots \\ \widetilde{v}_{2,l_\mathrm{S},m_2^{(2)}}^{(2)} \end{bmatrix}$$

is the vector of response values for sensor l_S and is obtained as follows:

1. Evaluate the vector of sensor responses

$$\mathbf{v}_{1,l_\mathrm{S}}^{(2)} = A_{l_\mathrm{S}}^{(2)*} + B_{l_\mathrm{S}}^{(2)*}\mathbf{y}_1^{(2)} + \mathbf{r}_{l_\mathrm{S}}^{(2)},$$

corresponding to the times $t_{1,i_1}^{(2)}$, $i_1 = 1, \ldots, m_1^{(2)}$, where

$$\begin{bmatrix} A_{l_\mathrm{S}}^{(2)*} \\ B_{l_\mathrm{S}}^{(2)*} \end{bmatrix} \sim \mathrm{N}\left(\begin{bmatrix} A_{l_\mathrm{S}}^{(2)} \\ B_{l_\mathrm{S}}^{(2)} \end{bmatrix}, \boldsymbol{\Sigma}_{l_\mathrm{S}}^{(2)} \right),$$

with

$$\mathbf{\Sigma}_{l_\mathrm{S}}^{(2)} = \begin{bmatrix} \left(\dfrac{u_\mathrm{r}\left(A_{l_\mathrm{S}}^{(2)}\right) A_{l_\mathrm{S}}^{(2)}}{100}\right)^2 & \dfrac{\mathrm{cov}_\mathrm{r}\left(A_{l_\mathrm{S}}^{(2)}, B_{l_\mathrm{S}}^{(2)}\right) A_{l_\mathrm{S}}^{(2)} B_{l_\mathrm{S}}^{(2)}}{100} \\ \dfrac{\mathrm{cov}_\mathrm{r}\left(A_{l_\mathrm{S}}^{(2)}, B_{l_\mathrm{S}}^{(2)}\right) A_{l_\mathrm{S}}^{(2)} B_{l_\mathrm{S}}^{(2)}}{100} & \left(\dfrac{u_\mathrm{r}\left(B_{l_\mathrm{S}}^{(2)}\right) B_{l_\mathrm{S}}^{(2)}}{100}\right)^2 \end{bmatrix},$$

and

$$\mathbf{r}_{l_\mathrm{S}}^{(2)} = \begin{bmatrix} r_{l_\mathrm{S},1}^{(2)} \\ \vdots \\ r_{l_\mathrm{S},m_1^{(2)}}^{(2)} \end{bmatrix},$$

with

$$r_{l_\mathrm{S},i_2}^{(2)} \sim \mathrm{N}\left(0, \left(\delta_{l_\mathrm{S}}^{(2)}\right)^2\right), \ i_2 = 1, \ldots, m_1^{(2)}.$$

2. For sensors that are faulty, the response values within the time intervals $\left[T_{\mathrm{F},l_\mathrm{F}}^{(2),1}, T_{\mathrm{F},l_\mathrm{F}}^{(2),2}\right]$ are obtained similarly to those in step 1, but using the parameters $C_{l_\mathrm{F}}^{(2)}, u_\mathrm{r}\left(C_{l_\mathrm{F}}^{(2)}\right), D_{l_\mathrm{F}}^{(2)}, u_\mathrm{r}\left(D_{l_\mathrm{F}}^{(2)}\right)$ and $\mathrm{cov}_r\left(C_{l_\mathrm{F}}^{(2)}, D_{l_\mathrm{F}}^{(2)}\right), \ l_\mathrm{F} = 1, \ldots, n_\mathrm{F}^{(2)}$.

3. Determine the times $t_{2,i_2}^{(2)}, \ i_2 = 1, \ldots, m_2^{(2)}$, at which the sensor response values are to be evaluated.

4. Evaluate the sensor response values $v_{2,l_\mathrm{S},i_2}^{(2)}$ corresponding to the times $t_{2,i_2}^{(2)}$, $i_2 = 1, \ldots, m_2^{(2)}$, by applying linear interpolation to the sensor responses $v_{1,l_\mathrm{S},i_1}^{(2)}$ corresponding to the times $t_{1,i_1}^{(2)}, \ i_1 = 1, \ldots, m_1^{(2)}$.

5. The sensor response values $v_{2,l_\mathrm{S},i_2}^{(2)}, \ i_2 = 1, \ldots, m_2^{(2)}$, are then quantized according to the values of $n_\mathrm{B}^{(2)}$ and $s^{(2)}$ (if present) to give values

$$\widetilde{v}_{2,l_\mathrm{S},i_2}^{(2)}, \ i_2 = 1, \ldots, m_2^{(2)}.$$

If no values have been provided for $n_\mathrm{B}^{(2)}$ and $s^{(2)}$, then

$$\widetilde{v}_{2,l_\mathrm{S},i_2}^{(2)} = v_{2,l_\mathrm{S},i_2}^{(2)}, \ i_2 = 1, \ldots, m_2^{(2)}.$$

6. For sensors that have missing data, the proportions $p_{\mathrm{M},l_\mathrm{M}}^{(2)}$ of data packets (a data packet is defined to be a group of data points that are sequential in time) chosen randomly within the time intervals $\left[T_{\mathrm{M},l_\mathrm{M}}^{(2),1}, T_{\mathrm{M},l_\mathrm{M}}^{(2),2}\right]$ have their values set to Not-a-Number (NaN), $l_\mathrm{M} = 1, \ldots, n_\mathrm{M}^{(2)}$.

5 Subsidiary measurand estimates and associated uncertainties

The array $\mathbf{Y}_2^{(2)}$ of subsidiary measurand estimates is given by

$$\mathbf{Y}_2^{(2)} = \begin{bmatrix} y_{2,1,1}^{(2)} & \cdots & y_{2,n_\mathrm{S}^{(2)},1}^{(2)} \\ \vdots & \ddots & \vdots \\ y_{2,1,m_2^{(2)}}^{(2)} & \cdots & y_{2,n_\mathrm{S}^{(2)},m_2^{(2)}}^{(2)} \end{bmatrix},$$

where

$$y_{2,l_\mathrm{S},i_2}^{(2)} = \frac{\widetilde{v}_{2,l_\mathrm{S},i_2}^{(2)} - A_{l_\mathrm{S}}^{(2)}}{B_{l_\mathrm{S}}^{(2)}}.$$

The array $\mathbf{U}_2^{(2)}$ of standard uncertainties associated with the subsidiary measurand estimates is given by

$$\mathbf{U}_2^{(2)} = \begin{bmatrix} u\left(y_{2,1,1}^{(2)}\right) & \cdots & u\left(y_{2,n_\mathrm{S}^{(2)},1}^{(2)}\right) \\ \vdots & \ddots & \vdots \\ u\left(y_{2,1,m_2^{(2)}}^{(2)}\right) & \cdots & u\left(y_{2,n_\mathrm{S}^{(2)},m_2^{(2)}}^{(2)}\right) \end{bmatrix},$$

where

$$\begin{aligned} u^2\left(y_{2,l_\mathrm{S},i_2}^{(2)}\right) &= \left(\frac{1}{B_{l_\mathrm{S}}^{(2)}}\right)^2 \left(\frac{u_\mathrm{r}\left(A_{l_\mathrm{S}}^{(2)}\right) A_{l_\mathrm{S}}^{(2)}}{100}\right)^2 + \left(\frac{y_{2,l_\mathrm{S},i_2}^{(2)}}{B_{l_\mathrm{S}}^{(2)}}\right)^2 \left(\frac{u_\mathrm{r}\left(B_{l_\mathrm{S}}^{(2)}\right) B_{l_\mathrm{S}}^{(2)}}{100}\right)^2 + \\ & \quad 2\left(\frac{y_{2,l_\mathrm{S},i_2}^{(2)}}{\left(B_{l_\mathrm{S}}^{(2)}\right)^2}\right) \left(\frac{\mathrm{cov}_\mathrm{r}\left(A_{l_\mathrm{S}}^{(2)}, B_{l_\mathrm{S}}^{(2)}\right) A_{l_\mathrm{S}}^{(2)} B_{l_\mathrm{S}}^{(2)}}{100}\right) + \left(\frac{\delta_{l_\mathrm{S}}^{(2)}}{B_{l_\mathrm{S}}^{(2)}}\right)^2. \end{aligned}$$