Calculation of estimates of observable output signal for examples 2A and 2B

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1 Introduction

This document describes the information that must be provided by the user to define the sensor responses to the observable output signal in the examples contained in MATLAB scripts `DataFusionSoftware_2A.m` and `DataFusionSoftware_2B.m`.

In particular, section 2 contains a list of all the numerical input that the user is expected to provide. For users who are interested, section 3 lists previously calculated information that is used when determining the sensor responses, while sections 4 and 5 provide full mathematical descriptions of how the sensor response values and estimates of the observable output signal (and associated uncertainties) are calculated, respectively, by the function `SecondOrderSystemSens.m`.

2 User input

The responses of the sensors to the observable output signal are defined by the following information provided by the user:

Sampling and quantization

- f_2, the sampling frequency (in Hz) [cell B10],
- (optional) n_{B}, the number of bits for quantization [cell B13],
- (optional) s, the saturation value [cell B14].

It is assumed that $f_2 < f_1$, i.e., the sampling frequency for the sensors is less than the sampling frequency used when calculating the 'true' measurand (section 3).

Behaviour of sensors

- $n_{\rm S}$, the number of sensors [cell B9 – filled in automatically from information in column C],
- $f_{l_{\rm S}},\ l_{\rm S} = 1, \ldots, n_{\rm S}$, the resonant frequencies (in Hz) [column D],
- $Q_{l_{\rm S}},\ l_{\rm S} = 1, \ldots, n_{\rm S}$, the Q-factors [column E],
- $\delta_{l_{\rm S}},\ l_{\rm S} = 1, \ldots, n_{\rm S}$, the levels of additive noise in the sensor output [column F].

Additional

- $f_{\min}$, the minimum frequency for approximating the (frequency-dependent) amplitude response of the sensors (in Hz) [cell B11],
- $f_{\max}$, the maximum frequency for approximating the (frequency-dependent) amplitude response of the sensors (in Hz) [cell B12].

Ideally, the interval $[f_{\min}, f_{\max}]$ should include the estimate $\tilde{f}_1$ of the frequency component of the measurand.

3 Additional input

The following information, previously calculated, is used:

- $t_{1,i_1},\ i_1 = 1, \ldots, m_1$, the times at which system responses are observed (corresponding to the sampling frequency f_1),
- $y_{{\rm O},i_1},\ i_1 = 1, \ldots, m_1$, the observed system responses.

4 Sensor response values

The array $\widetilde{\mathbf{V}}_2$ of sensor response values is given by

$$\widetilde{\mathbf{V}}_2 = [\widetilde{\mathbf{v}}_{2,1} \quad \ldots \quad \widetilde{\mathbf{v}}_{2,n_{\rm S}}]\,,$$

where

$$\widetilde{\mathbf{v}}_{2,l_{\rm S}} = \begin{bmatrix} \widetilde{v}_{2,l_{\rm S},1} \\ \vdots \\ \widetilde{v}_{2,l_{\rm S},m_2} \end{bmatrix}$$

is the vector of response values for sensor $l_{\rm S}$ and is obtained as follows:

1. Calculate

$$a_{l_{\mathrm{S}},0} = \frac{c_{l_{\mathrm{S}},0}\,(\delta t)^2 - 4c_{l_{\mathrm{S}},1}\delta t + 4}{c_{l_{\mathrm{S}},0}\,(\delta t)^2 + 4c_{l_{\mathrm{S}},1}\delta t + 4}, \quad a_{l_{\mathrm{S}},1} = \frac{2c_{l_{\mathrm{S}},0}\,(\delta t)^2 - 8}{c_{l_{\mathrm{S}},0}\,(\delta t)^2 + 4c_{l_{\mathrm{S}},1}\delta t + 4},$$

$$b_{l_{\mathrm{S}},0} = \frac{(\delta t)^2}{c_{l_{\mathrm{S}},0}\,(\delta t)^2 + 4c_{l_{\mathrm{S}},1}\delta t + 4}, \quad b_{l_{\mathrm{S}},1} = \frac{2\,(\delta t)^2}{c_{l_{\mathrm{S}},0}\,(\delta t)^2 + 4c_{l_{\mathrm{S}},1}\delta t + 4},$$

$$b_{l_{\mathrm{S}},2} = \frac{(\delta t)^2}{c_{l_{\mathrm{S}},0}\,(\delta t)^2 + 4c_{l_{\mathrm{S}},1}\delta t + 4},$$

   where

$$\omega_{l_{\mathrm{S}}} = 2\pi f_{l_{\mathrm{S}}}, \quad d_{l_{\mathrm{S}}} = \frac{-\omega_{l_{\mathrm{S}}}}{2Q_{l_{\mathrm{S}}}}, \quad c_{l_{\mathrm{S}},0} = \omega_{l_{\mathrm{S}}}^2 + d_{l_{\mathrm{S}}}^2, \quad c_{l_{\mathrm{S}},1} = -d_{l_{\mathrm{S}}} \quad \text{and} \quad \delta t = 1/f_1.$$

   Evaluate

$$w_{1,l_{\mathrm{S}},i_1} = \begin{cases} b_{l_{\mathrm{S}},2}\,y_{\mathrm{O},i_1}, & i_1 = 1, \\ \displaystyle\sum_{k=1}^{2} b_{l_{\mathrm{S}},k}\,y_{\mathrm{O},i_1-2+k} - a_{l_{\mathrm{S}},1}\,w_{1,l_{\mathrm{S}},i_1-1}, & i_1 = 2, \\ \displaystyle\sum_{k=0}^{2} b_{l_{\mathrm{S}},k}\,y_{\mathrm{O},i_1-2+k} - \sum_{k=0}^{1} a_{l_{\mathrm{S}},k}\,w_{1,l_{\mathrm{S}},i_1-2+k}, & i_1 = 3,\ldots,m_1. \end{cases}$$

   Calculate

$$v_{1,l_{\mathrm{S}},i_1} = w_{1,l_{\mathrm{S}},i_1} + r_{i_1}, \quad i_1 = 1,\ldots,m_1,$$

   where

$$r_{i_1} \sim \mathrm{N}\left(0, (\delta_{l_{\mathrm{S}}})^2\right).$$

2. Determine the times t_{2,i_2}, $i_2 = 1,\ldots,m_2$, at which the sensor response values are to be evaluated.

3. Evaluate the sensor response values v_{2,l_{S},i_2} corresponding to the times t_{2,i_2}, $i_2 = 1,\ldots,m_2$, by applying linear interpolation to the sensor responses v_{1,l_{S},i_1} corresponding to the times t_{1,i_1}, $i_1 = 1,\ldots,m_1$.

4. The sensor response values v_{2,l_{S},i_2}, $i_2 = 1,\ldots,m_2$, are then quantized according to the values of n_{B} and s (if present) to give values

$$\widetilde{v}_{2,l_{\mathrm{S}},i_2}, \ i_2 = 1,\ldots,m_2.$$

   If no values have been provided for n_{B} and s, then

$$\widetilde{v}_{2,l_{\mathrm{S}},i_2} = v_{2,l_{\mathrm{S}},i_2}, \ i_2 = 1,\ldots,m_2.$$

5 Observable output signal estimates and associated uncertainties

The array $\mathbf{Y}_2$ of observable output signal estimates is given by

$$\mathbf{Y}_2 = \begin{bmatrix} y_{2,1,1} & \cdots & y_{2,n_\mathrm{S},1} \\ \vdots & \ddots & \vdots \\ y_{2,1,m_2} & \cdots & y_{2,n_\mathrm{S},m_2} \end{bmatrix},$$

where

$$y_{2,l_\mathrm{S},i_2} = \frac{\widetilde{v}_{2,l_\mathrm{S},i_2}}{g_{l_\mathrm{S}}},$$

and the estimated gain g_{l_S} for sensor l_S is calculated as follows:

1. Determine n_E evenly spaced frequency values $f_{\mathrm{E},j}$ in the interval $[f_{\min}, f_{\max}]$.
2. For each frequency $f_{\mathrm{E},j},\ j = 1, \ldots, n_\mathrm{E}$, set

$$\omega_j = 2\pi f_{\mathrm{E},j},$$

and calculate

$$R_{l_\mathrm{S},j} = \sqrt{A_{l_\mathrm{S},j}^2 + B_{l_\mathrm{S},j}^2},$$

where

$$A_{l_\mathrm{S},j} = \frac{c_{l_\mathrm{S},0} - \omega_j^2}{(2c_{l_\mathrm{S},1}\omega_j)^2 + \left(c_{l_\mathrm{S},0} - \omega_j^2\right)^2} \quad \text{and} \quad B_{l_\mathrm{S},j} = \frac{-2c_{l_\mathrm{S},1}\omega_j}{(2c_{l_\mathrm{S},1}\omega_j)^2 + \left(c_{l_\mathrm{S},0} - \omega_j^2\right)^2}.$$

3. Set

$$g_{l_\mathrm{S}} = \mathrm{mean}\left(\{R_{l_\mathrm{S},j},\ j = 1, \ldots, n_\mathrm{E}\}\right).$$

The array $\mathbf{U}_2$ of standard uncertainties associated with the observable output signal estimates is given by

$$\mathbf{U}_2 = \begin{bmatrix} u\left(y_{2,1,1}\right) & \cdots & u\left(y_{2,n_\mathrm{S},1}\right) \\ \vdots & \ddots & \vdots \\ u\left(y_{2,1,m_2}\right) & \cdots & u\left(y_{2,n_\mathrm{S},m_2}\right) \end{bmatrix},$$

where

$$u\left(y_{2,l_\mathrm{S},i_2}\right) = \frac{\delta_{l_\mathrm{S}}}{g_{l_\mathrm{S}}}.$$